CREATING EQUITY AND EQUITY INDEX VOLATILITY SURFACES

IN REFINITIV® INSTRUMENT PRICING ANALYTICS (IPA)

Refinitiv® Instrument Pricing Analytics (IPA) offers a parametric representation of the equity and equity index volatility surfaces. This representation can be obtained independently for each group of option expiries (SVI) or globally for the surface across all available expiries and strikes (SSVI). The fitting algorithm is applied to a subset of option contracts selected from the entire universe of the available options for the underlying equity or equity index. In this paper we describe:

• How to create a request in IPA to generate a volatility surface
• How the options used to generate the volatility surface are selected and the IPA filters that control this selection
• How IPA generates the implied volatilities for the selected options
• The models IPA implements to fit the smiles for the selected options
One of the key steps in the generation of a volatility surface is the selection of the options that will be sent to the algorithm that will fit the smiles or the surface globally. There are two main objectives:

- Ensuring the consistency of the options used (e.g. exercise mode, option types)
- Ensuring the quality of the quotations (e.g. liquidity)

To guarantee that, the following pre-defined filters are applied:

- Options with missing reference data (expiry date, strike) are discarded.
- Duplicated options (same strike, expiry, call/put, exchange, exercise style) are removed. In this case, contracts with the highest open interest are kept.
- When an underlying has both American and European options underwritten, only the American ones are kept.
- For large data set (i.e., with more than 1,500 options), keep the options with a positive open interest, but only if this filter doesn’t reduce the size of the data set by more than 50%.
- Only the options with a valid price side and timestamp are kept in order to derive implied volatilities.
- When options are quoted on several exchanges, and if no exchange is specified in the request, only the exchange with the most data is kept. Also, exchanges with less than 15 options are ignored. If there is no exchange with more than 15 options, an error message explaining that there aren’t enough options to build the surface is returned.

This leads to a first data set to which additional filters may be applied, most of them being user-configurable:

- At-the-Money options can be removed based on a tolerance compared to the underlying price. The default value is 1%, but this can be adjusted with the parameter “atmToleranceIntervalPercent.” Also, only the options with a moneyness between 2% and 200% are kept. This later filter can’t be modified.
- Unless specifically requested with the parameters “useWeeklyOptions,” weekly options are discarded.
- Expiries with not enough strikes may be discarded using the “maturityFilter” parameter. The filter is based on the number of strikes in each expiry compared to the median number of strikes across all expiries and is controlled with the parameter “minOfMedianNbOfStrikesPercent.” Alternatively, a range for the expiries may be provided using the “minMaturity” and “maxMaturity” parameters.
- For a given expiry, volatilities that diverge too much from the median volatility can be discarded using the “strikeFilter” parameter.
- Finally, it’s also possible to identify the longest monotonous subsequence of calls or puts for each maturity and discard options that are not part of that sequence. This is controlled by the parameter “ensurePricesMonotonicity.”

Working with flexibility in the selection of the option contracts thanks to user-defined filters

---

Working with flexibility in the selection of the option contracts thanks to user-defined filters
Generating a series of implied volatilities from Refinitiv option prices, dividends and analytics

The calibration algorithms (SVI or SSVI) run on the implied volatilities of the options in the data set filtered as described in the previous section. If the implied volatilities are provided by the contributors, they can be used directly. Otherwise, they are implied from the option premia, the underlying spot prices quoted at the same time as the premia, and the dividend forecast. This switch is controlled by setting the value of the parameter "inputVolatilityType" to "Quoted" or "Implied."

In addition, the surface can be generated from intraday quotes (for the most liquid instruments), end-of-day quotes or the settlement values provided by the contributors. This is controlled by the parameter "timeStamp" with the values “Close,” “Settle,” “Default.” Finally, when “Close” or “Default” is used, the parameter “priceSide” can be used to control the side of the quote (“Mid,” “Bid,” “Ask”). By default, and if possible, a “Mid” value is used.

Implied forwards, dividends and risk-free rates

Critical to the calculation of the implied volatility are the implied equity or equity index forward prices. They are determined from the spot price selected above, the future dividends over the option period and the risk-free rate for that period.

This risk-free rate is interpolated on the default zero coupon curve of the currency of the underlying. The curve used is the default one for that currency and is usually built from a mix of deposits, forward rate agreements (FRAs), futures and LIBOR-based swaps.

For equities, the present value of the forecasted dividend payments collected by the Refinitiv Content team is subtracted from the equity spot price. For indices, a dividend yield representing the forecasted dividend of the constituents of the index is used instead. In this case, the forward price is computed as follows:

$$\text{forward}(T) = \text{Spot}(t) \times \frac{e^{-\text{divyield}(T) \times (T-t)}}{e^{-\text{riskfree}(T) \times (T-t)}}$$
Introducing a goodness of fit indicator to assess the quality of the fitted smiles

Finally, the algorithm generates a parametric representation of the equity and equity index volatility surface based on the list of the selected options and their implied volatilities. This representation can be obtained independently for each group of option expiries (SVI) or globally for the surface across all available expiries and strikes (SSVI). The reader can refer to “The SVI and SSVI algorithm description” to get a detailed description of the SVI and SSVI algorithms.

For the purpose of this paragraph, the SVI minimization formula is represented below to show that in addition to the parameters $a, b, \rho, \sigma, m$, the formula needs the forward price of the underlying ($f$) at the option expiry $T$ and the implied volatility of these options ($\sigma_{\text{market}}$). The SSVI formula has the same dependency on the forward price and the market volatility. Therefore, the selection of the options used in the calibration, and the accuracy in the calculations of the underlying forward price and the implied volatilities, is critical to a good representation of these smiles.

$$\min_{(a,b,\rho,\sigma,m)} \sum_{i=1}^{N} \left( (a, b, \rho, \sigma, m), \ln \left( \frac{K_i}{f}, T \right) - \sigma_{\text{market}}(K_i, T) \right)^2$$

where $T$ is the expiry of the selected listed options and $K_i$ their individual strikes. $f$ represents the forward price of the underlying instrument and $(a,b,\rho,\sigma,m)$ the SVI parameters with the following constraints:

$$a > 0, \quad \sigma \geq 0, \quad \rho \in [-1,1], \quad a \geq 0, \quad b \leq \frac{4}{(1+|\rho|)T}$$

Goodness of fit

In addition to the surface, as the fit of the SVI or the SSVI is not perfect by design, IPA returns an indicator of the quality of the fit (goodness of fit) that could indicate a potential calibration issue or an issue in the data. The indicator is provided for each smile. It is simple, as the formula below illustrates:

$$g = 1 - \frac{\sum_{i=1}^{N} |\sigma_i^\text{model} - \sigma_i^\text{market}|}{\sum_{i=1}^{N} |\sigma_i^\text{model} - \sigma_i^\text{ATM}|}$$

A goodness of fit close to 1 means a good fit to the given market data. This indicator provides an easy assessment of the calibration to the option market data.
A parameterized API to control the generation of the surfaces and their display

The surface can be retrieved using the Refinitiv Instrument Pricing Analytics (IPA). The construction of the surface is controlled by parameters that are defined in a JSON request file whose structure is described at https://developers.refinitiv.com/en/api-catalog/refinitiv-data-platform/refinitiv-data-platform-apis/documentation#ipa-volatility-surfaces-eti.

Linear interpolation on the variance

In addition to the parameters available to control how the options are filtered and the smiles fit, it is also possible to choose the tenors and the strikes that will be returned in the surface. By default, the surface will contain a smile per expiry tenor and will return a volatility for the 80%, 85%, 90%, 95%, 97.5%, 100%, 102.5%, 105%, 110%, 115%, 120% moneyness levels or equivalent strikes depending on the choice made in the "xAxis" or "yAxis" parameters.

However, the parameters "xValues" and "yValues" in the "surfaceOutput" section may be used to specify the tenors and strikes to be returned. In this case the interpolation or extrapolation along the time axis may be required and is described below:

\[ \sigma^2 t = a_1^2 t_1 + \frac{a_2^2 t_2 - a_1^2 t_1}{t_2 - t_1} (t - t_1) \]

The extrapolation uses the \( \alpha \)-interpolation method described by Fabien Le Floch and summarized below:

\[ \sigma_{0,b,p,\alpha,m}^2 = a t + b t_n \left[ \rho(x - m) + \sqrt{(x - m)^2} \right] \text{for} \ t \geq t_n \]

Appendix – The SVI and SSVI algorithm description

This appendix briefly describes the two algorithms that may be used to fit the volatility smiles.

SVI parametrization

The SVI approach fits a separate SVI parametrization for each expiry option expiry.

\[ \sigma_{a,b,\rho,\sigma,m}^2 = a + b \left[ \rho(x - m) + \sqrt{(x - m)^2 + \sigma^2} \right] \]

Where:

\[ x = \ln \left( \frac{K}{f} \right) \]
\[ f = \text{forward price} \]

The parameters have the following constraints:

\[ a > 0, \quad \sigma \geq 0, \quad \rho \in [-1, 1], \quad a \geq 0, \quad b \leq \frac{4}{(1 + |\rho|)T} \]

And the following impact:

- Increasing \( a \) increases the general level of variance (vertical translation of the smile)
- Increasing \( b \) increases the slopes of both the call and put wings, tightening the smile
- Increasing \( \rho \) decreases (increases) the slope of the left (right) wing, a counterclockwise rotation of the smile
- Increasing \( m \) translates the smile to the right
- Increasing \( \sigma \) reduces the at the money (ATM) curvature of the smile

The function \( \sigma^2(x) \) at a given expiry date has a minimum at the point \((x^*, v^*)\) with:

\[ x^* = m - \frac{\rho \sigma}{\sqrt{1 - \rho^2}} \]
\[ v^* = a + b \rho \sqrt{1 - \rho^2} \]

Each of the SVI parameters are calibrated at each expiry by solving the following minimization problem:

\[ \min_{\{a,b,\rho,\sigma,m\}} \sum_{l=1}^{N} \left[ \sigma \left[ \{a,b,\rho,\sigma,m\}, \ln \left( \frac{K_l}{f}, T \right) - \sigma_{\text{market}}(K_l, T) \right] \right]^2 \]

where \( T \) is a fixed listed expiry and \( K_1 \ldots K_N \) are the listed option strikes for that maturity.
Surface SVI (SSVI) parametrization

The smile-by-smile approach where a separate SVI is fitted at each option expiry usually fits each smile well but is at times not robust enough against bad data or outliers. Instead, Sebastien Gurrieri\(^2\) proposes the following parametric form with 11 parameters \((s_0, s_0, t, \alpha, \beta, x_0, \lambda_0, \delta)\) to model the entire volatility surface with a global SVI parametrization:

\[
\sigma^2(x, T) = v_T^* + b_T \left[ n_T(x - x_T^*) + n_T(x - x_T^*) - \lambda_T \right]
\]

Where:

\[
n_T(x) = \sqrt{x^2 - 2\rho x \lambda + \lambda^2}
\]

\[
\rho_T = \rho
\]

\[
x_T^* = x_0^* - \rho (\lambda_T - \lambda_0)
\]

\[
b_T = \alpha T^{\beta - 1}
\]

\[
\lambda_T = \lambda_0 + \frac{\gamma}{\delta + 1} T^\delta + 1
\]

\[
v_T^* = \frac{\alpha y (1 - \rho^2)}{\beta + \delta + 1} T^{\beta + \delta} - B_T + s_0^* - \frac{1}{T} (B_T + B_T + s_0^* + s_0^*) \left( e^{-T/\tau} - 1 \right)
\]

With the following constraints:

\[
\rho \in [-1, 1], \alpha \geq 0, \beta \in ]0, 1[, \lambda_0 \geq 0, \gamma_0 \geq 0, \delta > -1, \tau > 0, B \geq 0
\]

For a given underlying instrument or index, the SSVI parameters are fitted against the whole list of options as a two-step process:

- The initial value of the minimization is found with a differential evolution.
- The solution is then found using a Levenberg-Marquardt minimization algorithm.

---

\(^2\) S. Gurrieri, A Class of Term Structures for SVI Implied Volatility (October 25, 2010).
Available at SSRN: http://ssrn.com/abstract=1779463
Refinitiv is one of the world's largest providers of financial markets data and infrastructure, serving over 40,000 institutions in approximately 190 countries. It provides leading data and insights, trading platforms, and open data and technology platforms that connect a thriving global financial markets community – driving performance in trading, investment, wealth management, regulatory compliance, market data management, enterprise risk and fighting financial crime.